For all questions, answer (E) NOTA means none of the above answers is correct. All numbers on this test are real numbers. All functions on this test have domains and ranges that are subsets of the real numbers.

1. **A**) Use the definition of limit. The correct answer is A

2. **B**) Since the value at $f(0)$ is a removable discontinuity, $\lim\limits_{x\to a} f(x)=0$ for all real number a. It is easy to see $f(f(x))=\begin{cases}1, x\neq 0\\ 0, x=0\end{cases}$. Therefore, $\lim\limits_{x\to a} f(f(x))=1$ for all real number a. The correct answer is B.

3. **C**) $\lim\limits_{x\to 2} f(x)=\lim\limits_{x\to 2}(x^2+1)=5$. The correct answer is C.

4. **D**) $\lim\limits_{x\to 1} f(x)=\lim\limits_{x\to 1}(\frac{x-1}{x^2+1})=\frac{0}{2}=0$. The correct answer is D.

5. **B**) Use L'Hospital's rule, we have $\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to\infty}\frac{\sqrt{4x^3+2x^2-5x}}{\sqrt{x^3}-x-\sqrt{4x}}=\lim\limits_{x\to\infty}\frac{\sqrt{4+\frac{2}{x}-\frac{5}{x^2}}}{1-\frac{1}{\sqrt{x}}-\sqrt{\frac{4}{x^2}}}=2$. The correct answer is B.

6. **D**) Use L'Hospital's rule, we have $\lim\limits_{x\to 2} f(x)=\lim\limits_{x\to 2}\frac{x^4-2x-12}{x^5-2x^2-24}=\lim\limits_{x\to 2}\frac{4x^3-2}{5x^4-4x}=\frac{5}{12}$. The correct answer is D.

7. **C**) Use L'Hospital's rule, we have
$\lim\limits_{x\to 0}\frac{2\sin x-\sin 2x}{x-\sin x}=\lim\limits_{x\to 0}\frac{2\cos x-2\cos 2x}{1-\cos x}=\lim\limits_{x\to 0}\frac{-2\sin x+4\sin 2x}{\sin x}=\lim\limits_{x\to 0}\frac{-2\cos x+8\cos 2x}{\cos x}=6$. The correct answer is C.

8. **D**) Use L'Hospital's rule, we have

$\lim\limits_{x\to 0}\frac{3^{2x}-1}{2x}=\lim\limits_{x\to 0}\frac{2\cdot 3^{2x}\ln 3}{2}=\ln 3$. The correct answer is D.

9. **C**) Use L'Hospital's rule, we have

$\lim\limits_{x\to 0}\frac{\ln(4\sin x+1)}{2x}=\lim\limits_{x\to 0}\frac{4\cos x}{2(4\sin x+1)}=2$. The correct answer is C.

10. **D**) Since $\lim_{x\to 0^+}\frac{3}{1-3^{-\frac{1}{x}}}=\lim_{y\to -\infty}\frac{3}{1-3^{y}}=3$. The correct answer is D.

11. **A**) Since $\lim_{x\to 0}x=0$ and $|\sin\frac{1}{x}|\le 1$, we have $0=\lim_{x\to 0}-|x|\le\lim_{x\to 0}x\sin\frac{1}{x^2}\le\lim_{x\to 0}|x|=0$ The correct answer is A

12. **A**) Since $\lim_{y\to 0}y=0$ and $0\le\frac{x^2}{x^2+y^2}\le 1$, we have $0=\lim_{y\to 0}-|y|\le\lim_{x\to 0}\frac{2x^2y}{x^2+y^2}\le\lim_{x\to 0}|y|=0$.The correct answer is A

13. **A**) Since $f'(x)=3x^2+1\ge 1$for any real number . The correct answer is A

14. **C**) If $a<2$, then from $\lim_{x\to a}f(x)=f'(a)$, we have $a+2=1\Rightarrow a=-1$. If $a>2$, then from $\lim_{x\to a}f(x)=f'(a)$, we have $2a=a^2\Rightarrow a=2$. In this case, it is easy to see . $\lim_{x\to 2^+}f(x)=f'(2^+)$ but $\lim_{x\to 2^-}f(x)\ne f'(2^-)$ The correct answer is C

15. **C**) Let $\lim_{x\to 1}f(x)=\lim_{x\to 1}x+1=2$. The correct answer is C.

16. **B**) $\lim_{x\to 1}\sin^{-1}\left(\frac{1-\sqrt{x}}{1-x}\right)=\lim_{x\to 1}\sin^{-1}\left(\frac{1-\sqrt{x}}{(1-\sqrt{x})(1+\sqrt{x})}\right)=\lim_{x\to 1}\sin^{-1}\left(\frac{1}{(1+\sqrt{x})}\right)=\lim_{x\to 1}\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$. The correct answer is B.

17. **D**) Since

$$f(x)=x^4+\sin x^2+e^{\cos x}+5\Rightarrow f(0)=e+5$$
$$f'(x)=4x^3+2x\cos x^2-\sin xe^{\cos x}\Rightarrow f'(0)=0$$
$$f''(x)=12x^2+2\cos x^2-4x^2\sin x^2-\cos xe^{\cos x}+\sin^2 xe^{\cos x}\Rightarrow f''(0)=2-e$$

Therefore $g(0)=e+5+2-e=7$. The correct answer is D

18. **C**) Since $f(x)=\int_0^{x^2}(\sin t^2+e^{\sin^{-1}t})dt$, we have

$f'(x)=2x\sin x^4+2xe^{\sin^{-1}x^2}$

$f''(x)=8x^4\cos x^4+2\sin x^4+2e^{\sin^{-1}x^2}+\frac{4x^2}{\sqrt{1-x^4}}e^{\sin^{-1}x^2}$

$f''(0)=2$. The correct answer is C.

19. **C**) $\lim_{n\to\infty}\sum_{i=2}^{n}\frac{4}{3(i^2-1)}=\lim_{n\to\infty}\sum_{i=2}^{n}\frac{2}{3}(\frac{1}{i-1}-\frac{1}{i+1})=\lim_{n\to\infty}\frac{2}{3}(\frac{3}{2}-\frac{1}{n}-\frac{1}{n+1})=1$. The correct answer is C

20. **D**)From $f(x)=\dfrac{12(x-1)^{\frac{2}{3}}\sqrt{x+1}}{(1-x^2)^3}=-12(x-1)^{-\frac{7}{3}}(x+1)^{-\frac{5}{2}}$, we have

$f'(x)=-12(-\dfrac{7}{3}(x-1)^{-\frac{10}{3}}(x+1)^{-\frac{5}{2}}-\dfrac{5}{2}(x-1)^{-\frac{7}{3}}(x+1)^{-\frac{7}{2}})$. Therefore

$f'(0)=-12\cdot(-\dfrac{7}{3}+\dfrac{5}{2})=-2$. The correct answer is D

21. **E**) From $x^3+y^2=80$, we have $x^3+y^2\Rightarrow 3x^2+2yy'=0\Rightarrow y'=-\dfrac{3x^2}{2y}$ and

$y''=-\dfrac{6xy-3x^2y'}{2y^2}=-\dfrac{3x}{y}+\dfrac{3x^2y'}{2y^2}=-\dfrac{3x}{y}-\dfrac{9x^4}{4y^3}$

$y'-y''=\dfrac{3x}{y}+\dfrac{9x^4}{4y^3}-\dfrac{3x^2}{2y}\Rightarrow (y'-y'')|_{(-4,-4)}=3-\dfrac{9}{4}4+\dfrac{3}{2}4=3+9-6=6$. The correct answer is E

22. **C**)Let $\lim\limits_{x\to\infty}\cos\dfrac{a}{x}=\lim\limits_{y\to\infty}\cos y=1$ .The correct answer is A.

23. **C**) $y^5\sin x^2-\cos(x-y)=0\Rightarrow 5y^4y'\sin x^2+2xy^5\cos x^2+(1-y')\sin(x-y)=0\Rightarrow \dfrac{dy}{dx}|_{(0,\frac{\pi}{2})}=1$ .The correct answer is C

24. **B)** $\lim\limits_{x\to 0}\dfrac{\cos^{2017}x-1}{\cos x-1}=\lim\limits_{x\to 0}(1+\cos x+\cos^2 x+\cdots\cos^{2016}x)=2017$. The correct answer is B.

25. **A)** $\lim\limits_{x\to\frac{\pi}{2}}(1+\cot x)^{\frac{1}{\cos x}}=\lim\limits_{x\to\frac{\pi}{2}}(1+\dfrac{\cos x}{\sin x})^{\frac{1}{\cos x}}=\lim\limits_{x\to\frac{\pi}{2}}(1+\dfrac{\cos x}{\sin x})^{\frac{\sin x}{\cos x}\cdot\frac{1}{\sin x}}=\lim\limits_{x\to\frac{\pi}{2}}e^{\frac{1}{\sin x}}=e$. The correct answer is A.

26. **C)** $y=\sqrt{x+a\sqrt{x+a\sqrt{x+a\sqrt{x+\cdots}}}}\Rightarrow y^2=x+a\sqrt{x+a\sqrt{x+a\sqrt{x+\cdots}}}\Rightarrow y^2=x+ay$.

$2yy'=1+ay'\Rightarrow(2y-a)y'=1\Rightarrow y'=\dfrac{1}{2y-a}$. The correct answer is C.

27. **B)** $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\dfrac{1}{i+n}=\int_0^1\dfrac{1}{x+1}dx=\ln(x+1)|_0^1=\ln 2$. The correct answer is B.

28. **A)** $\lim\limits_{x\to\pi}\sin(x+\sin x)=\sin(\lim\limits_{x\to\pi}(x+\sin x))=\sin\pi=0.$. The correct answer is A.

29. **C)** $f(x)=2x\sin x\Rightarrow f'(x)=2\sin x+2x\cos x\Rightarrow f'(x)|_{x=\frac{\pi}{2}}=2$. The equation of tangent line is

$y-f(\dfrac{\pi}{2})=f'(\dfrac{\pi}{2})(x-\dfrac{\pi}{2})\Rightarrow y=2x$. The correct answer is C.

30. 30. **B**) $f(x)=x\sin^{-1}x+\sqrt{1-x^2}\Rightarrow f'(x)=\sin^{-1}x+\dfrac{x}{\sqrt{1-x^2}}-\dfrac{x}{\sqrt{1-x^2}}=\sin^{-1}x$.

31. $f'(x)|_{x=\frac{1}{2}} = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$ The correct answer is B.